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Optimizing campaign sizing policies: an application to a real-life setting

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Optimizing campaign sizing policies: an application to a real-life setting

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Abstract

This paper presents an integrated production inventory model that enables to capture the trade-offs between average inventory, production capacity and customer service level in a semi-process industry setting. The model includes different features that are specific for such a setting, such as differences in reactor yield and quality requirements across products, the need for cleaning reactors when switching between product types, and the requirement to produce products in campaign sizes that are an integer multiple of the reactor's batch size. The model can be used to support midterm planning procedures. In this paper, we illustrate the application of the model to real-life data of two product families at a large specialty chemicals company, which for reasons of confidentiality is further referred to as Company C.

Keywords: Queueing, campaign sizing, (semi)process industries

1. Introduction

For over 60 years, Company C has been designing and supplying specialty products for the cosmetics, pharmaceutical and vaccine markets, as well as some industrial niches. As of 2009, the company was hit by the consequences of the financial crisis: forecast accuracies were decreasing due to increased uncertainty in demand, and at the same time there was a strong pressure to control working cash and operating costs. Despite this difficult environment,

[1] Both co-authors are members of the Supply Chain Management Department of a large specialty chemicals company, which provided the real-life data to perform experiments in Section 3 of the paper. For reasons of confidentiality, details on the company cannot be revealed.

Company C still had to meet the high expectations of its customers, who themselves faced the same difficulties.

A particularly critical issue was to ensure enough resources (finished goods stock versus reactor capacity) to respond to the actual demand and maintain the service level. The challenge was to continuously find the balance between service level, stock value, and operating costs due to extra capacity. The model described in this paper provides the basis of the methodology and the company tools that were eventually implemented for that purpose. It combines queueing theory and inventory theory to reflect the dynamic behaviour of Company C's operations. To tune the balance between capacity, inventory and service level, the model uses the campaign sizes of the products (which refer to the amount of batches of the same product produced consecutively on a reactor).

The model is fed by many operational data (forecasts, delivery conditions, manufacturing and supply conditions in all plants) and supports several strategic planning processes at a corporate level, including CAPEX and S&OP. As such, it supports decision making at the midterm planning level. It also provides the Supply Chain team with quick updates on parameter values such as safety stocks, re-order points, utilization rate, expected lead-times, expected service level etc.

The use of queueing theory to model a semi-process industry setting is in fact quite unusual. Apart from earlier work by Carlson and Felder (1992) and Van Nieuwenhuyse et al. (2007), academic research in this area seems inexistent. Issues related to campaign sizing, campaign scheduling and product cycling tend to be studied from a deterministic perspective (e.g. Rajaram and Karmarkar (2004), Rajaram and Karmarkar (2002), Dobson (1987), Elmaghraby (1978), Fleischmann (1990)). The few research efforts aimed at modelling process industry settings from a stochastic perspective have predominantly focused on discrete-event simulation (e.g., Felder et al. 1983, Felder et al. 1985).

For decision support, however, the queueing approach provides distinct advantages. It allows to incorporate the impact of several managerial decisions (e.g., campaign sizing, outsourcing) to fine-tune system performance through optimization and what-if analyses. Moreover, runtimes are very short: even for complicated, real-life systems, different what-if scenarios can be analysed in a matter of seconds. Last but not least, it enables to reflect the impact of different sources of randomness on operational performance, yielding more realistic estimates of capacity usage, replenishment lead times and average inventory levels. This provides a big advantage against deterministic approaches. In the literature, the use of queueing models to improve business intelligence and support decision making at the midterm planning level is referred to as Advanced Resource Planning (see e.g. Van Nieuwenhuyse et al. 2011).

This research builds on the model proposed in Van Nieuwenhuyse et al. (2007), which we adapt to take into account shift constraints, yield factors, and additional time delays for quality control and transportation. It allows to evaluate the impact of the company's current campaign sizing policies on average inventory, customer service and reactor capacity. Moreover, the model was augmented with an optimization procedure (genetic algorithm) to determine the optimal campaign sizes, in view of minimizing total average inventory. The resulting model and optimization procedure were originally coded in MATLAB, and are discussed in Section 2. Section 3 shows the model's results for two of Company C's product families (family 2 and family 3). Finally, Section 4 summarizes the main conclusions from this project.

2. Methodology

Figure 1 gives a schematic overview of Company C's operations. Products tend to be produced to stock; this stock is depleted by incoming customer orders. As the inventory position reaches the reorder point, a replenishment order is triggered and the manufacturing order enters a queue waiting for the reactor to become available. The order is processed and conditioned on the reactor; afterwards, it is sent to quality control. When approved, the order is transported to the warehouse.

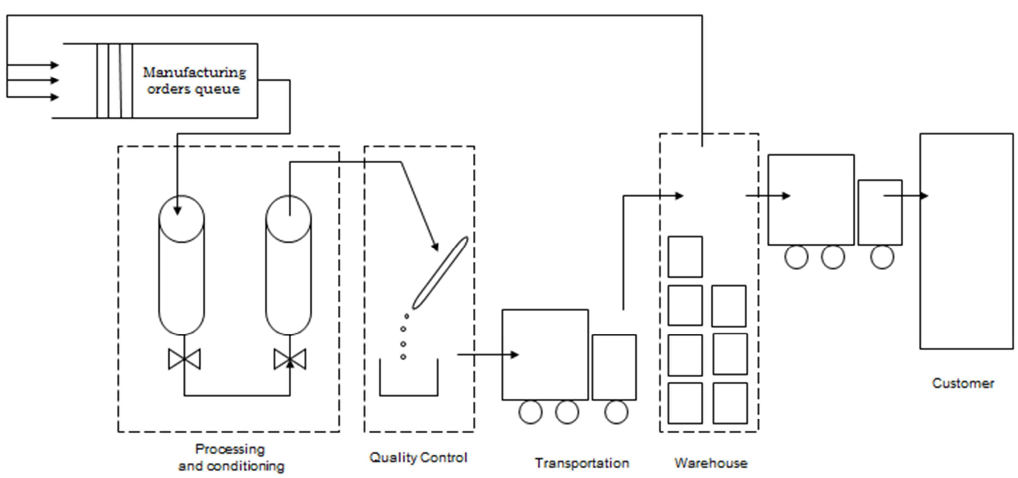


Figure 1: Schematic overview of Company C's operations

As such, Company C's operational system behaves as an *integrated production/inventory system*: the stock control policies at the central warehouse impact production decisions (as they determine the frequency and amount of replenishment orders), while the resulting production performance (in terms of average and variability of the replenishment lead times) impacts stock management and customer service (e.g. safety stock requirements and customer service levels). The challenge is to optimally exploit the interplay between the inventory system and the production system, taking into account different mediating factors (such as the availability of the reactors, cleaning and conditioning times, etc).

In reality, Company C's product portfolio consists of different product families. As each product family is produced on reactors dedicated to that particular family (i.e., reactors are not "shared" across families), production and inventory decisions can be analysed for each product family separately. Consequently, the unit of analysis in the mathematical model is the product family. The decision variables are the campaign sizes of the product types. In the chemical industry, these campaign sizes indeed have a large impact on operational performance. Setups are needed when the reactor switches from one product type to another (e.g. for cleaning), which implies that short campaigns reduce the effective capacity of a reactor. On the other hand, long campaigns imply long replenishment lead times for finished goods inventories, and consequently increase the average inventory required to provide a target customer service level.

The next subsection details how the operational setting of Figure 1 is captured in the model, and introduces notation.

2.1. Notation

In general, each product family consists of K product types (index k = 1 to K) and is produced on a set of reactors (index r = 1 to R) dedicated to that family. Within each family, different product subsets PS_r might be distinguished, grouping those products that need to be processed on the same reactor r. Subsets within a family tend to be disjoint (i.e., each product type k can be processed on a single reactor r and hence belongs to a single subset PS_r).

Figure 2 shows an illustration of the integrated production/inventory system for a given product family, assuming all products are treated by a single reactor r. The reactor has a given batch size B_r, which is a technical characteristic and refers to the amount of raw material that can be put into the reactor. The actual yield of the reactor may, however, vary widely across the product types produced; denoting the yield percentage of product type k on reactor r by $y_{k,r}$, the actual amount of end product obtained for product k on reactor r (which will be referred to as the reactor yield) equals $B_{k,r} = y_{k,r} * B_r$. Yield factors can be smaller than 100% (for instance in case of evaporation, distillation or waste elimination). It may exceed 100% when finished product density is above 1.

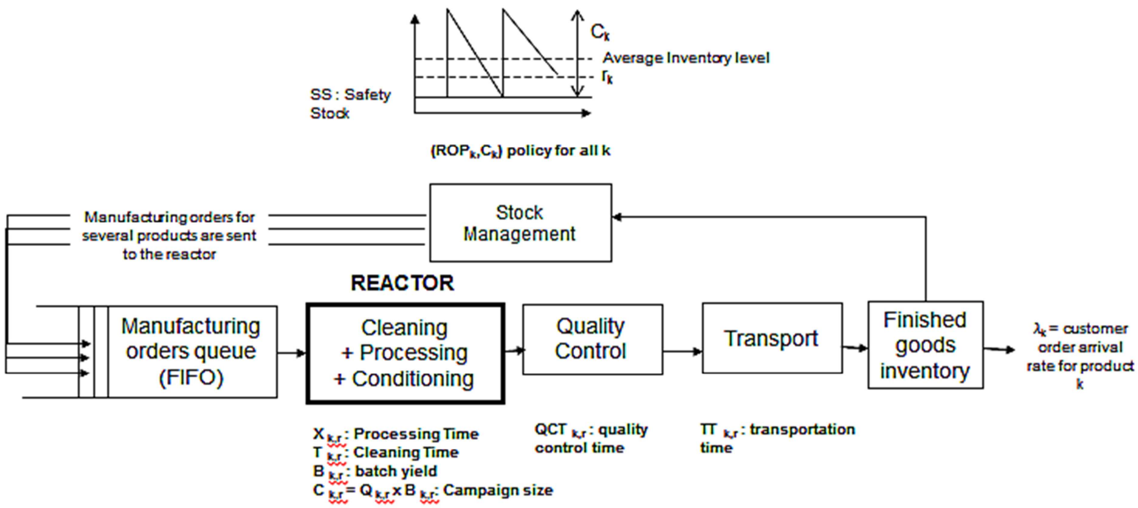


Figure 2: Integrated production/inventory system for arbitrary product family

Every batch of product type k that is allocated to reactor r needs to be processed and conditioned. Two configurations are possible :

1. The finished product is conditioned directly in the reactor, which consequently remains unavailable until conditioning ends. Afterwards, the reactor is cleaned if a different product type is to be processed. In this case the batch processing time includes the conditioning time.
2. The finished product is stored in an intermediate tank after processing, and is conditioned afterwards in another workshop. In this case, conditioning time is not included in the batch processing time.

The resulting batch processing time is a general random variable, denoted by $X_{k,r}$. A "changeover time" (i.e., cleaning time) is required when switching production between different product types. The time needed to clean reactor r after production of product k is also a random variable, denoted by $T_{k,r}$; during this time, the reactor remains unavailable for processing.

We model the customer order pattern for each product type k within a given product family by a Poisson distribution with arrival rate λ_k. For modelling purposes, customer orders are assumed to have unit size (in practice, the average order size is at least greater or equal than the smallest product packaging, e.g. 30kg). The average interarrival time of customer orders of type k will be denoted by $\overline{Y}_k$ ($\lambda_k = \frac{1}{\overline{Y}_k}$). The finished goods inventories of all product types k are individually managed according to a continuous review (ROP_k,$C_{k,r}$) policy: when the inventory position of product type k reaches level ROP_k, a replenishment order of size $C_{k,r}$ is triggered and sent to reactor r. $C_{k,r}$ refers to the campaign size to be produced for product k on reactor r, and needs to be an integer multiple of the reactor yield $B_{k,r}$:

$$C_{k,r} = Q_{k,r} * B_{k,r}$$

$Q_{k,r}$ is referred to as the *campaign size multiplier*, and can be any integer number ≥ 1. In practice, campaign sizes can be chosen by management. Customer orders that arrive while the product is out of stock, join the system as *backorders*.

The queueing discipline among campaigns at a given reactor is assumed to be FIFO. Note that this queue is not a physical queue; hence, it makes sense to assume that the capacity of the queue is infinite.

All batches produced in a given campaign are subject to quality control before being transported to the central warehouse. Sometimes, for unknown reasons, finished product is out of specifications: the campaign is then rejected. Currently, the probability of rejection is not taken into account in the model, due to a lack of representative data. The quality control time is referred to by a general random variable $QCT_{k,r}$; the transportation time is referred to as $TT_{k,r}$. In general, the transportation time will tend to be independent of the product type and the reactor. The quality control time can vary from one product to another, depending on its properties and quality requirements (e.g. pharmaceutical products tend to have longer quality control time due to high quality requirements). As there is no waiting time involved for quality control and transportation, both time components are modelled as pure delay times in the model.

Due to the competition for capacity, the replenishment lead times of campaigns allocated to the same reactor are interdependent. The arrival and service characteristics of the campaigns for products within PS_r will impact the expected waiting time in queue $EW_{q,r}$ (and variance $VW_{q,r}$) experienced at reactor r. As evident from Figure 2, these arrival and service characteristics are in turn influenced by the inventory control parameters of the products in PS_r (r_k and C_k for every individual product type k).

Moreover, the reactors at Company C operate under differing modes: “3x8” mode (which means that the reactor is available 4.5 days per week, 24 hours per day, followed by 2.5 days of planned downtime), “4x8” mode (availability of 5.5 days per week with 1.5 day of planned downtime) or “5x8” mode (reactor operates 7 days per week). The limited availability in the “3x8” and “4x8” system further impacts the capacity of the reactor, which in turn impacts the expected waiting times of campaigns.

The challenge is to optimally exploit the interplay between the inventory system and the production system, taking into account the capacity of the reactor under different operating modes. More specifically, we aim to determine the campaign size multipliers $Q_{k,r}$ in such a way that the minimum required customer service level for each individual product type k (CSL_k^*) is preserved, while the total average inventory level across all product types in the family is minimized.

For ease of reference, Table 1 gives an overview of the notation. For any random variable Z, $\overline{Z}$ refers to its average, s_Z^2 to its variance and c_Z^2 to its squared coefficient of variation ($c_Z^2 = \frac{s_Z^2}{\overline{Z}^2}$).

r	= reactor index ($r=1,\ldots,R$)
k	= product type index ($k=1,\ldots,K$)
$y_{k,r}$	= yield factor for product type k on reactor r
$B_{k,r}$	= reactor yield of product type k on reactor r
$Q_{k,r}$	= campaign size multiplier of product type k on reactor r
$C_{k,r}$	= campaign size of product type k on reactor r = $Q_{k,r} * B_{k,r}$
$T_{k,r}$	= setup time for campaign of product type k on reactor r
$X_{k,r}$	= processing time for a batch of product type k on reactor r
$QCT_{k,r}$	= quality control time for campaign of product type k produced on reactor r
$TT_{k,r}$	= transportation time to move product of type k produced on reactor r to final stock
CSL_k^*	= minimum required customer service level for product type k
ROP_k	= reorder point for product type k
PS_r	= subset of products produced on reactor r

Table 1: Overview of notation

2.2 Estimating replenishment lead times and customer service levels

As finished goods inventories are managed according to a continuous review policy, the customer service level for any arbitrary product type k in the system (CSL_k) is determined by the probability of not running out of stock during the replenishment lead time. Obviously, this equals the probability that the demand during lead time for product type k ($DDLT_k$) does not exceed the reorder point ROP_k:

$$CSL_k = \text{Prob}(DDLT_k \leq r_k) \tag{1}$$

To approximate the probability distribution of $DDLT_k$, we need to approximate the probability distribution of the replenishment lead time for a campaign of type k on reactor r. Following Lambrecht et al. (1998), we approximate this lead time by a lognormal distribution with density function:

$$f_{W_{k,r}}(t) = \frac{1}{t * \sqrt{2\Pi\sigma_{k,r}^2}} * \exp\left\{\frac{-(\ln(t) - \mu_{k,r})^2}{2\sigma_{k,r}^2}\right\} \tag{2}$$

where the shape parameter $\sigma_{k,r}$ and the scale parameter $\mu_{k,r}$ are linked to the mean ($E(W)_{k,r}$) and the variance ($V(W)_{k,r}$) of the lognormal distribution (e.g. Law and Kelton, 2000):

$$\sigma_{k,r}^2 = \ln(1 + SCV(W)_{k,r})$$

$$\mu_{k,r} = \ln\left(\frac{E(W)_{k,r}}{\sqrt{1 + SCV(W)_{k,r}}}\right)$$

$$SCV(W)_{k,r} = \frac{V(W)_{k,r}}{[E(W)_{k,r}]^2}$$

Both $E(W)_{k,r}$ and $V(W)_{k,r}$ can be approximated by considering each reactor as a *GI/G/1* queueing system, as discussed in Van Nieuwenhuyse et al. (2007). For ease of reference, the structure of the resulting queueing model is discussed in Appendices A-B; as evident from these expressions, the impact of the campaign sizing decisions is reflected in both $E(W)_{k,r}$ and $V(W)_{k,r}$.

The probability distribution of $DDLT_k$ is then approximated by the convolution of $f_{W_{k,r}}(t)$ and the Poisson arrival process of product type k:

$$P(DDLT_k == x) = \int_{t=0}^{+\infty} e^{-\lambda_k t} * \frac{(\lambda_k t)^x}{x!} * \left[\frac{1}{t * \sqrt{2\Pi\sigma_{k,r}^2}} * \exp\left\{\frac{-(\ln(t) - \mu_{k,r})^2}{2\sigma_{k,r}^2}\right\}\right] dt \tag{3}$$

Using expressions (1) and (3), the customer service level achieved for product type k (CSL_k) given a reorder point ROP_k is obtained as:

$$
\begin{aligned}
& CSL_k \\
& = \sum_{i=0}^{ROP_k} P(DDLT_k == i) \qquad (4) \\
& = \sum_{i=0}^{ROP_k} \int_{t=0}^{+\infty} \frac{e^{-\lambda_k t} * (\lambda_k t)^i}{i!} * \left[\frac{1}{t * \sqrt{2\Pi\sigma_{k,r}^2}} * \exp\left\{ \frac{-(\ln(t) - \mu_{k,r})^2}{2\sigma_{k,r}^2} \right\} \right] dt
\end{aligned}
$$

Expression (4) can be evaluated using numerical integration for any reorder point ROP_k.

2.3. Optimization problem

As mentioned in section 2.1, the objective of the company is to determine the campaign size multipliers $Q_{k,r}$ in such a way that the minimum required customer service level for each individual product type k (CSL_k^*) is preserved, while the average inventory for the family is minimized. The optimization problem can then be formally stated as:

$$
\begin{aligned}
& \underset{Q_{k,r}}{Min} \quad I = \sum_{r=1}^{R} \left[\sum_{k \in PS_r} \left(ROP_k - E(DDLT)_k \right) + \sum_{k \in PS_r} \frac{Q_{k,r}}{2} BS_{k,r} \right] \\
& s.t. \quad \text{(a) } CSL_k \geq CSL_k^*, \forall k \\
& \qquad \text{(b) } lb_{k,r} \leq Q_{k,r} \leq ub_{k,r}, \forall k, r \qquad (5) \\
& \qquad \text{(c) } \rho_r < 1, \forall r \\
& \qquad \text{(d) } Q_{k,r} \geq 1 \text{ and integer}, \forall k, r
\end{aligned}
$$

The average inventory is expressed in kg rather than in euros, as the production unit cost tends to be identical for all products belonging to the same product subset (and subsets are disjoint). The first term of the objective function reflects the expected safety stock, while the second term reflects the expected cycle stock. By definition, the safety stock for an arbitrary product k equals the difference between its reorder point and its expected demand during lead time, while the cycle stock depends on the campaign sizes used (as $C_{k,r} = Q_{k,r} BS_{k,r}$). Note that in the expression for safety stock, the reorder point of any arbitrary product k is inherently determined by service level constraint (a) ($CSL_k \geq CSL_k^*$); as such, it also depends directly on the chosen campaign size multipliers $Q_{k,r}$ as discussed in section 2.2. Moreover, the decision variables $Q_{k,r}$ may be required to satisfy upper and lower bound constraints as specified in (b). The remaining constraints ensure that the solution is feasible (reactors may not be overloaded, and campaign size multipliers need to be integers).

The objective function as well as constraints (a) and (c) are nonlinear functions of the decision variables. Consequently, problem (5) is an integer constrained nonlinear programming problem. For product families with a limited number of products and tight bound constraints on the multipliers (as is the case for Family 3, see Section 3), the optimum might be found using exhaustive search. For more complex settings, a heuristic approach can be used (we opted for a genetic algorithm, implemented in MATLAB).

3. Results and discussion

The model described in Section 2 was applied to two of Company C's product families: family 2 and family 3.

Family 2 consists of 16 product types, which are produced in a single reactor. The demand data and process time data are shown in Appendix C1. As evident from the data, many products in this family are currently produced in campaigns of 7 (sometimes 8) batches. The reactor size B_r is 10 tons; however, the reactor yield $B_{k,r}$ tends to differ (e.g. product 1 is evaporated, resulting in a reactor yield of 5500kg). Last year, the reactor operated mostly in 3x8 or 4x8 mode; as of March 2011, the company switched to 5x8 mode. The required customer service level for all products in this family is 95%. There are technical bound constraints on the campaign size multipliers: product types 5 to 16 need to be produced in campaigns of 5 batches at least, as this is the minimum amount of product required before conditioning can start. The upper bound on the campaign size multiplier equals 8 for all products (except product type 8, which can be produced in campaigns of at most 7 batches) .

Family 3 consists of only 6 product types, which again share a single reactor. The demand and process time data are shown in appendix C2. Products 1 to 5 are strategic and require a service level of 98%. Product 6 is a commodity; the required service level is 90%. As evident from the data, the current campaign size for product 6 is notably larger than for the strategic products (8 batches per campaign), which implies that it is replenished rather infrequently. This reactor usually operates in 3x8 mode, which implies that the reactor is often saturated. For this reason, the company considers to outsource production of product 6. For technical reasons, there are constraints on the multipliers for products 1 through 4: $Q_{1,r}$ and $Q_{3,r}$ should be at most 4 while $Q_{2,r}$ and $Q_{4,r}$ should be at most 3.

For both families, cleaning times and processing times can be assumed deterministic (note however that in 3x8 and 4x8 mode, the limited availability of the reactors alters both average and variability of these time components as discussed in Appendix A). Quality control times and transportation times are assumed to be uniformly distributed between bounds that deviate +/- 20% of their expected values (which are shown in Appendix C).

3.1. Results for Family 2

As shown in Table 2, the optimal campaign size multipliers coincide with the lower bounds both in 4x8 and 5x8 mode ($Q^{*}_{k,r} = 1$ for k = 1 to 4 and $Q^{*}_{k,r} = 5$ for k = 5 to 16). Currently, only the campaign size of product 1 is optimal. For all other product types, campaign sizes should be reduced. These results indicate that capacity is relatively abundant for this family, allowing the company to perform relatively frequent setups.

k	$Q_{k,r}$ current	$Q^{*}_{k,r}$	
		4x8	5x8
1	1	1	1
2	2	1	1
3	2	1	1
4	2	1	1
5	7	5	5
6	7	5	5
7	8	5	5
8	7	5	5
9	7	5	5
10	7	5	5
11	7	5	5
12	7	5	5
13	7	5	5
14	7	5	5
15	7	5	5
16	7	5	5

Table 2: Optimal versus current campaign sizing policies for product family 2

Figure 3 summarizes the results for the estimated inventory (top pane), and reactor utilization (bottom pane). Currently, the reactor runs in 4x8 mode, meaning that it is unavailable (inactive) for 21% of the time.

The model estimates that with the current operating mode and the current campaign sizing policy, an average inventory level of 564.11 tons is required in order to maintain a 95% customer service level for all product types. The utilization of the reactor is at 64.9% (meaning that it is busy for 64.9 % of its available time). Maintaining 4x8 mode but switching to the optimal campaign sizes leads to an expected average inventory level of 400.84 tons, a 29%

decrease. As shown in Figure 3, this is largely attributable to a decrease in cycle stock (-31%). The total replenishment lead times are slashed by 9.64% on average[2], leading to a decrease in safety stock of approximately 15%. The reactor utilization (ρ_r) rises to 70.9%.

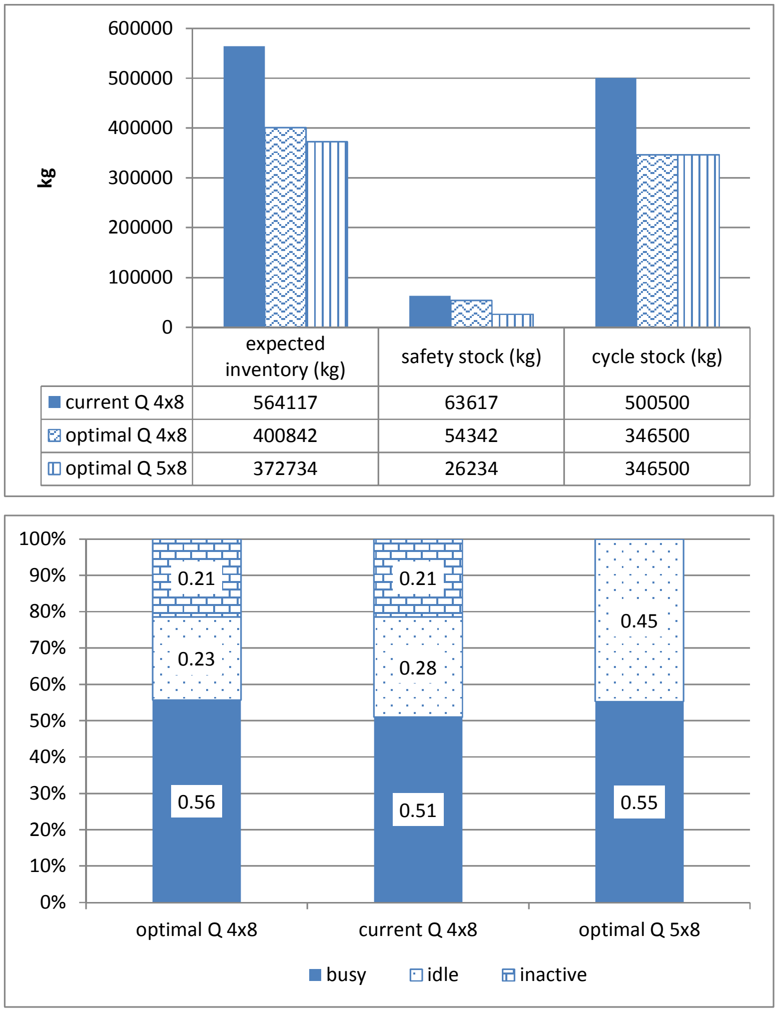


Figure 3: Results for expected inventory and reactor utilization, family 2

An effort to increase capacity by switching to 5x8 mode further cuts safety stock (approximately by half). As shown in Figure 3, this reduces expected inventory, though the additional impact is relatively small. At the same time, the reactor's utilization would shrink to 55% of its available time. Consequently, it might be more cost-effective for the company to invest in reducing quality control times and transportation times to further reduce expected inventory, rather than to increase the number of operating hours on the reactor. Interestingly,

[2] The lead times through the reactor are slashed by 20.26% on average. As the replenishment lead time contains quality control time and transportation time, which cannot be influenced through the campaign sizing policy, the percentage reduction in total lead time is less pronounced.

the company recently launched an internal project aiming to reduce the average quality control time to less than 4 working days. Moreover, the company no longer waits for the quality control results to ship product to the warehouse: the product is transported as soon as possible after conditioning ends. Though this introduces the risk of having to ship product back to the plant, this new practice is perceived as efficient as the risk of rejection is low.

3.2. Results for Family 3

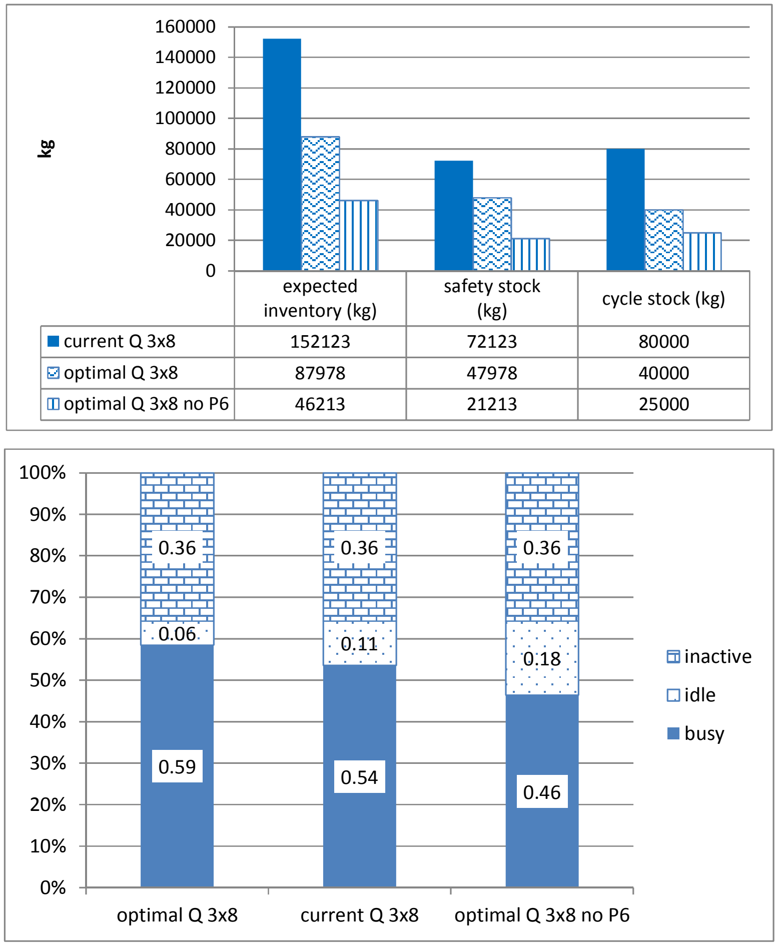
Table 3 shows the optimal campaign size multipliers for family 3, both for the scenario with the current product mix and the scenario in which the non-strategic product 6 is outsourced. The reactor of family 3 operates in 3x8 mode.

k	3x8 with current $Q_{k,r}$	$Q^*_{k,r}$	
		3x8 with current mix	3x8 with P6 outsourced
1	4	2	1
2	1	1	1
3	1	1	1
4	1	1	1
5	1	1	1
6	8	2	1

Table 3: Optimal versus current campaign sizing policies for product family 3

When product 6 is kept in portfolio, the current policy can be maintained for products 2 to 5 while campaign sizes for product 1 and 6 should be reduced to 2 batches. As detailed in Figure 4, this leads to an average inventory of 87.98 tons (as opposed to 152.12 tons[3] for the current policy), of which 40 tons cycle stock (80 tons with the current policy). Consequently, the impact of the optimal policy is rather dramatic: expected inventory is reduced by 42%, cycle stock is cut by half and safety stock is cut by 33%. As illustrated in the bottom pane of Figure 4, the utilization of the reactor reaches 91% of its available time (as opposed to 83% with the current policy).

[3] According to company data, the total stock for this family in the period May 2010-October 2010 averaged 146 tons.



<u>Figure 4:</u> Results for expected inventory and reactor utilization, family 3

Outsourcing the non-strategic product 6 allows to reduce all campaign sizes to one single batch. The utilization of the reactor drops to 72% of its available time. The expected inventory can be reduced to 46.21 tons (a 70% reduction compared to the current situation).

Despite these promising results, the implementation of the optimal policy in practice will be a long-term effort. After further investigation, a number of “hidden” constraints emerged for this family, which hamper the desired increase in changeovers. A first constraint is related to personnel: the cleaning is performed by teams of operators, which tend to be allocated to more than one reactor. Consequently, campaign switching is not only constrained by reactor availability but also by operator availability. A second constraint relates to environmental requirements, as an increase in changeovers might increase the amount of waste rejection. Bypassing this constraint would require an increase in the reprocessing cost before waste release. Furthermore, some raw materials are supplied in tanks which must be emptied at once.

Consequently, campaigns need to be scheduled such that the same product (or another which requires the same raw material) is produced until all raw material has been consumed. A decrease in campaign sizes is likely to complicate this scheduling effort, as multiple campaigns will be required to empty the raw material supply.

Even though the immediate applicability is limited, the model results are insightful for the company, as they allow the organization to explore hidden system constraints, and tackle these progressively contingent on their leverage.

4. Conclusions

This paper has presented a queueing model that enables semi-process industries to adequately model the trade-offs between capacity, average inventory and customer service level. The decision variables used to fine-tune these trade-offs are the campaign sizes of the products; these are indeed key variables in a semi-process setting, as they impact the frequency of setups (e.g. for cleaning) on the reactors, and hence impact reactor capacity.

Company C currently uses the model to support midterm planning and decision making processes (such as CAPEX and S&OP). In their experience, the model proves its value primarily by evaluating the impact of the current campaign sizing policies (e.g. on related safety stock levels and utilization rates), and exposing "hidden" constraints that prevent the introduction of the optimal campaign sizes. As such, it acts as an "eye"-opener, triggering initiatives to improve operational procedures and remove these constraints.

As the model is generic, it can be applied in other semi-process settings as well. One of the main advantages of the model is that it includes the impact of different sources of randomness (such as customer order patterns, uncertainties on quality control and transportation times, increases in variability due to limited reactor availability) on operational performance. This yields more realistic estimates of capacity usage, replenishment lead times and average inventory levels, giving it a distinct advantage over deterministic campaign sizing approaches. Given the short runtimes, it is particularly well suited to support decision making at the midterm level.

In future work, we plan to further extend the model in order to reflect the impact of product portfolios which consist of a mix of MTO (make-to-order) and MTS (make-to-stock) products. Furthermore, we aim to include the impact of (partial) subcontracting of production.

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Appendix A: Description of the queueing model

Each reactor is modelled as a multi-product *GI/G/1* queueing system[4]. The parameters of the individual arrival and service processes on each reactor are aggregated into a single aggregate arrival and service process, as explained next.

Assuming a campaign size multiplier $Q_{k,r}$ (≥ 1), the average arrival rate of campaigns of product type $k \in PS_r$ at reactor r is given by $l_{k,r}$:

$$l_{k,r} = \frac{\lambda_k}{Q_{k,r} * B_{k,r}} \qquad \text{(A.1)}$$

Note that this arrival rate is impacted by the yield factor $y_{k,r}$ (as $B_{k,r} = y_{k,r}*B_r$): ceteris paribus, a lower yield factor triggers more frequent campaign replenishments. The aggregate arrival rate of campaigns at reactor r equals:

$$l_r = \sum_{k \in PS_r} l_{k,r} \qquad \text{(A.2)}$$

As the arrival process of product type k at reactor r is Poisson, the squared coefficient of variation of the interarrival times for campaigns can be calculated as:

[4] Note that the grouping of customer orders into campaign sizes causes the arrival process of campaigns at the reactor to be Erlang distributed, while the processing and cleaning times are generally distributed.

$$ca_{k,r}^2 = \frac{1}{Q_{k,r} * B_{k,r}} \tag{A.3}$$

The squared coefficient of variation of the aggregate interarrival time of campaigns at this reactor can be approximated by (Whitt, 1983):

$$ca_r^{\;2} \approx w\left(\sum_{k \in PS_r} \frac{l_{k,r}}{l_r} ca_{k,r}^2\right) + 1 - w \tag{A.4}$$

where the weight w is given by:

$$w = \frac{1}{\left[1 + 4(1-\rho_r)^2 (v-1)\right]} \tag{A.5}$$

with

$$v^{-1} = \sum_{k \in PS_r} \frac{(l_{k,r})^2}{l_r^2} \tag{A.6}$$

and ρ_r as defined in expression (A.7) below.

The average aggregate campaign processing time on reactor r is calculated as a weighted average of the individual average campaign processing times:

$$\frac{1}{\mu_r} = \sum_{k \in PS_r} \frac{l_{k,r}}{l_r}\left[\overline{T}_{k,r} + Q_{k,r}\overline{X}_{k,r}\right] \tag{A.7}$$

where the weights reflect the relative importance of the campaign arrivals of the different product types, and μ_r refers to the aggregate processing rate of campaigns at reactor r. The squared coefficient of the aggregate campaign processing times is approximated by:

$$\begin{aligned} cs_r^{\;2} &= \left[\sum_{k \in PS_r} \frac{l_{k,r}}{l_r} E(T_{k,r} + Q_{k,r} X_{k,r})^2 \mu_r^{\;2}\right] - 1 \\ &= \frac{\mu_r^{\;2}}{l_r} * \left[\sum_{k \in PS_r} l_{k,r} * (\overline{T}_{k,r} + Q_{k,r}\overline{X}_{k,r})^2 * (cs_{k,r}^2 + 1)\right] - 1 \end{aligned} \tag{A.8}$$

In this expression, $cs_{k,r}^2$ refers to the squared coefficient of the campaign processing time for product k on reactor r:

$$cs_{k,r}^2 = \frac{s_{Tk,r}^2 + Q_{k,r} s_{Xk,r}^2}{(\overline{T}_{k,r} + Q_{k,r}\overline{X}_{k,r})^2}$$

The effective traffic intensity ρ_r of a reactor r is given by:

$$\rho_r = \frac{l_r}{\mu_r} = \sum_{k=1}^{K} l_{k,r}\left[\overline{T}_{k,r} + Q_{k,r}\overline{X}_{k,r}\right] \tag{A.9}$$

and is dependent on the campaign sizes $Q_{k,r}$. For large campaign sizes, $l_{k,r}$ tends to zero and the impact of the setup times on ρ_r is negligible. For small campaign sizes, ρ_r increases significantly due to the impact of the setup times. In order to preserve system stability, ρ_r should be strictly smaller than unity for all reactors r: $\rho_r < 1, \forall\, r$.

The different operating modes of the reactor ("3x8", "4x8" and "5x8", as discussed in Section 2.1) impact the processing characteristics of campaigns at that reactor. This impact is reflected in our model by introducing an availability factor A, which represents the fraction of time that the reactor is available (which equals 0.64 in 3x8 mode, 0.78 in 4x8 mode and 1 –i.e., full availability– in 5x8 mode)[5]. The (deterministic) downtimes in 3x8 and 4x8 mode are then taken into account by adjusting the average and variance of the cleaning and processing times on the reactor (Hopp and Spearman 2000; the notation x⟵y denotes that x is replaced by y):

$$
\begin{aligned}
\overline{T}_{k,r} &\leftarrow \frac{\overline{T}_{k,r}}{A} \\
\overline{X}_{k,r} &\leftarrow \frac{\overline{X}_{k,r}}{A} \\
s^2_{Tk,r} &\leftarrow \frac{s^2_{Tk,r}}{A^2} + \frac{(1-A)}{A}\overline{T}_{k,r} m_d \\
s^2_{Xk,r} &\leftarrow \frac{s^2_{Xk,r}}{A^2} + \frac{(1-A)}{A}\overline{X}_{k,r} m_d
\end{aligned}
\qquad \text{(A.10)}
$$

The notation m_d refers to the expected downtime per cycle (which equals 60 hours in 3x8 mode, and 36 hours in 4x8 mode). Note that the operating modes only impact cleaning and batch processing characteristics; they do not influence quality control times and transportation times.

Lead time approximations

The expected replenishment lead time of a campaign of product type k on reactor r can be obtained as:

$$E(W)_{k,r} = E(W_q)_r + \overline{T}_{k,r} + Q_{k,r} * \overline{X}_{k,r} + \overline{QCT}_{k,r} + \overline{TT}_{k,r} \qquad \text{(A.10)}$$

The term $E(W_q)_r$ stands for the average waiting time in front of the reactor, and can be approximated in a G/G/1 system by the Kraemer-Lagenbach Belz formula (Kraemer and Lagenbach-Belz (1976), Lambrecht et al. (1998)):

[5] In the 3x8 system, the reactors operate 108 hours per week, yielding an availability of 108/168=0.64. The availability in 4x8 mode equals 132/168=0.78.

$$E(W_q)_r = \frac{\rho_r^2 (ca_r^2 + cs_r^2)}{2l_r(1-\rho_r)} \exp\left\{ \frac{-2(1-\rho_r)(1-ca_r^2)^2}{3\rho_r (ca_r^2 + cs_r^2)} \right\} \quad \text{(A.11)}$$

as, in our setting, $ca_r^2 < 1$. Note that this approximation is independent of the product type *k*.

In the literature, it is commonly assumed that all lead time components are independent (see Lambrecht et al. (1998), Vandaele (1996)), such that the variance of the replenishment lead time can be approximated by:

$$V(W)_{k,r} = V(W_q)_r + s_{Tk,r}^2 + Q_{k,r} s_{Xk,r}^2 + s_{QCTk,r}^2 + s_{TTk,r}^2 \quad \text{(A.12)}$$

$V(W_q)_r$, stands for the variance of the waiting time spent in queue. It can be approximated using the formula of Whitt (1983), which is given in Appendix B.

Appendix B: Expression for $V(W_q)$

According to Whitt (1983), $V(W_q)$ can be written as[6]:

$$V(W_q) = [E(W_q)]^2 c_{Wq}^2$$

where

$c_{Wq}^2 \quad = \dfrac{c_D^2 + 1 - \sigma}{\sigma}$ represents the SCV of the waiting time;

$\sigma \quad = \rho + (ca^2 - 1)\rho(1-\rho)h(\rho, ca^2, cs^2)$ represents the probability of delay ($P(W_q>0)$)

$$h(\rho, ca^2, cs^2) = \frac{1 + ca^2 + \rho\ cs^2}{1 + \rho(cs^2 - 1) + \rho^2(4ca^2 + cs^2)} \quad \text{if } ca^2 \le 1$$

$$h(\rho, ca^2, cs^2) = \frac{4\rho}{ca^2 + \rho^2(4ca^2 + cs^2)} \quad \text{if } ca^2 \ge 1;$$

$c_D^2 = 2\rho - 1 + \dfrac{4(1-\rho)d_s^3}{3(cs^2+1)^2}$ denotes the SCV of the conditional waiting time, i.e. the waiting time given that the server is busy;

$d_s^3 = 3cs^2(cs^2 + 1) \quad$ if $cs^2 \ge 1$

$\quad = (2cs^2 + 1)(cs^2 + 1)\quad$ if $cs^2 < 1$

[6] For ease of notation, we omit the subscript *r* in the expressions.

Appendix C: demand and processing time data per product family

Appendix C.1: Product family 2

k	r	Current $Q_{k,r}$	$\overline{T}_{k,r}$ (hr)	$\overline{X}_{k,r}$ (hr)	$\overline{QCT}_{k,r}$ (hr)	$\overline{TT}_{k,r}$ (hr)	$B_{k,r}$ (kg)	λ_k (kg/week)
1	1	1	12	40	164	72	5500	876
2	1	2	12	24	164	72	11000	5269
3	1	2	12	24	164	72	12000	4154
4	1	2	12	32	164	72	12000	1465
5	1	7	12	14	164	72	10500	1250
6	1	7	12	14	164	72	10500	1431
7	1	8	12	14	164	72	12000	2692
8	1	7	12	14	164	72	13000	752
9	1	7	12	14	164	72	10500	2335
10	1	7	12	14	164	72	10500	5612
11	1	7	12	14	164	72	10500	817
12	1	7	12	14	164	72	11000	904
13	1	7	12	14	164	72	11000	1629
14	1	7	12	14	164	72	11000	560
15	1	7	12	14	164	72	10000	4442
16	1	7	12	14	164	72	10000	9231

Appendix C.2: Product family 3

k	r	Current $Q_{k,r}$	$\overline{T}_{k,r}$ (hr)	$\overline{X}_{k,r}$ (hr)	$\overline{QCT}_{k,r}$ (hr)	$\overline{TT}_{k,r}$ (hr)	$B_{k,r}$ (kg)	λ_k (kg/week)
1	1	4	10	20	100.8	72	10000	11858
2	1	1	10	40	100.8	72	10000	3447
3	1	1	10	20	100.8	72	10000	1778
4	1	1	10	40	100.8	72	10000	3557
5	1	1	10	20	100.8	72	10000	274
6	1	8	10	14	100.8	72	10000	13754